Lexical Analysis

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Version of 10:58 PM 15-Feb-2022

Regular Expression Notation

- We will develop a notation – called *regular expressions* – to describe languages that can be built from an alphabet (letters, numbers, and other symbols) with certain operations applied – potentially recursively applied

- Operators are
  - Union – designated by infix | (vertical bar)
  - Concatenation – designated by sequential subexpressions
  - Closure – designated by a postfix superscript of either * or +

Alphabet & Strings

- An *alphabet* is any finite set of symbols
- A *string* over an alphabet is a finite sequence of symbols drawn from that alphabet
- $|s|$ is the length of string *s*, *i.e.,* the number of symbols in string *s*
- ε is the string of length zero, *i.e.,* the *empty string*

Language, Union, and Concatenation

- Assume L is a *language*
  - L is any countable set of strings over some fixed alphabet
  - Includes the empty set, syntactically well-formed C programs, all grammatically correct English sentences
  - This definition of *language* does not ascribe any meaning to strings in the language
- Union of languages
  - Identical to the set operation of union
  - A language formed from the **union** of two languages is composed of all strings in either the first language or the second language
- Concatenation of languages
  - A language formed from the **concatenation** of two languages is composed of all strings formed by taking a string from the first language and a string from the second language, in all possible ways, and concatenating them

Closure

- Kleene Closure or, simply, Closure
  - The set of strings that can be created by concatenating L **zero** or more times
    - Denoted by $\mathbf{L^*}$
  - L^0 (*i.e.,* concatenation of L zero times) is defined to be { ε }
  - L^i , for $i > 0$, is defined to be $L^{i-1} L$
- Positive Closure
  - Same as the Kleene Closure, but without L^0
    - The set of strings that can be created by concatenating L **one** or more times
    - Denoted by $\mathbf{L^+}$
  - ε is not in L^+ unless ε is in L itself

Regular Expression

- Σ is the alphabet
- L(*r*), where *r* is a regular expression, is the language denoted by *r*
- ε is a regular expression
- L(ε) is {ε}, that is, the language whose sole member is the empty string
- If *a* is a symbol in Σ, then
  a is a regular expression and
  L(*a*) = {*a*}
- For regular expressions *r* and *s*,
  - (*r*) is a regular expression denoting L(*r*)
  - (*r*)|(*s*) is a regular expression denoting L(*r*) U L(*s*)
  - (*r*)(*s*) is a regular expression denoting L(*r*)L(*s*)
  - (*r*)* is a regular expression denoting (L(*r*))*

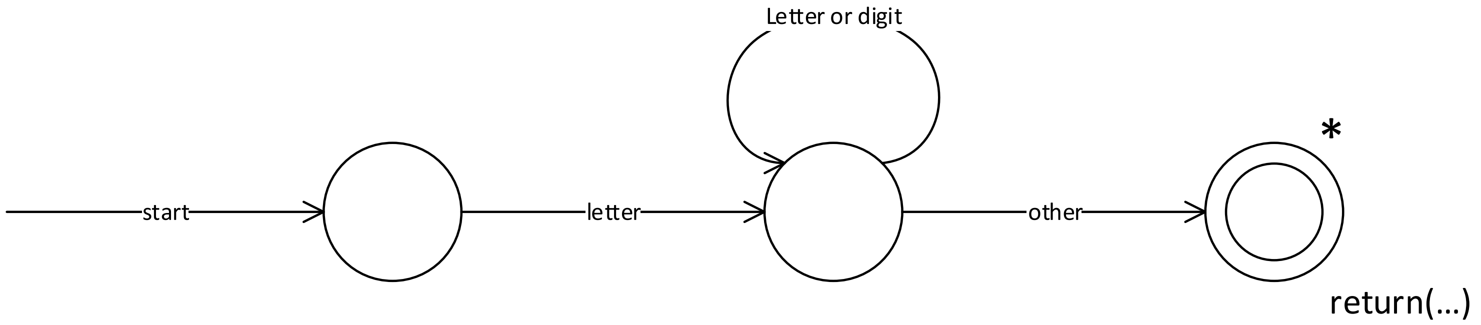
Precedence of Regular Expression Operators

- Highest precedence: Closure (or unary *)
- Next highest precedence: Concatenation
- Lowest precedence: Union or Alternation (or |)
- All operators are left associative

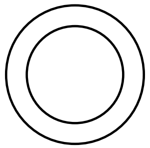
Algebraic Laws for Regular Expressions

- $r \mid s = s \mid r$ — $\mid$ is commutative
- $r \mid (s \mid t) = (r \mid s) \mid t$ — $\mid$ is associative
- $r(st) = (rs)t$ — concatenation is associative
- $r(s \mid t) = rs \mid rt$ — concatenation distributes over $\mid$
- $(s \mid t)r = sr \mid tr$ — concatenation distributes over $\mid$
- $\varepsilon r = r\varepsilon = r$ — ε is the identity for concatenation
- $r^* = (r \mid \varepsilon)^*$ — ε is guaranteed in a closure
- $r^{**} = r^*$ — * is idempotent

Transition Diagrams



Designated start state



Accepting (final) state

Edges labeled with symbol or set of symbols

* Retract one position (symbol)

Nondeterministic Finite Automata (NFA)

- A finite set of states *S*.
- A set of input symbols Σ, the *input alphabet*. We assume that ε, which stands for the empty string, is never a member of Σ.
- A *transition function* that gives, for each state, and for each symbol in Σ U {ε} a set of *next states*.
- A state s_0 from *S* that is distinguished as the *start state* (or *initial state*).
- A set of states *F*, a subset of *S*, that is distinguished as the *accepting states* (or *final states*).

Nondeterministic Finite Automata (NFA) compared to Deterministic Finite Automata (DFA)

- NFA
    - No restrictions on the labels on edges
        - The same symbol can label several edges out of the same state
        - ε, the empty string, is also a possible label
- DFA
    - For each state, and for each symbol of its input alphabet, there can be exactly one edge with that symbol leaving that state
    - No edge can be labeled with ε, the empty string
- Either an NFA or a DFA can be represented by a *transition graph*

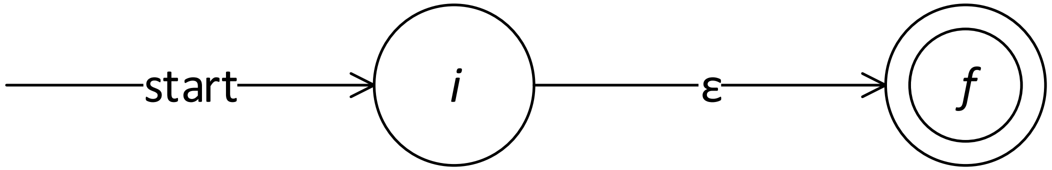
Construction of an NFA from a Regular Expression

- Apply the McNaughton-Yamada-Thompson algorithm

- Apply the algorithm on constituent subexpressions

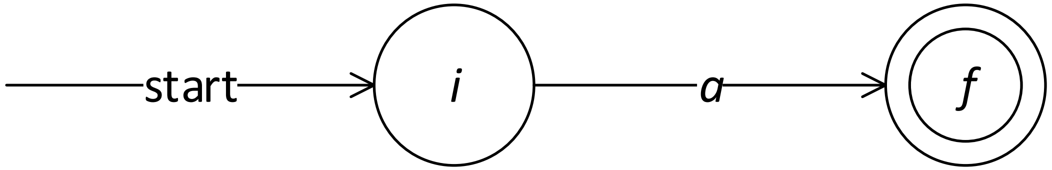
Construction of an NFA: expression **ε**

- For regular expression r = ε



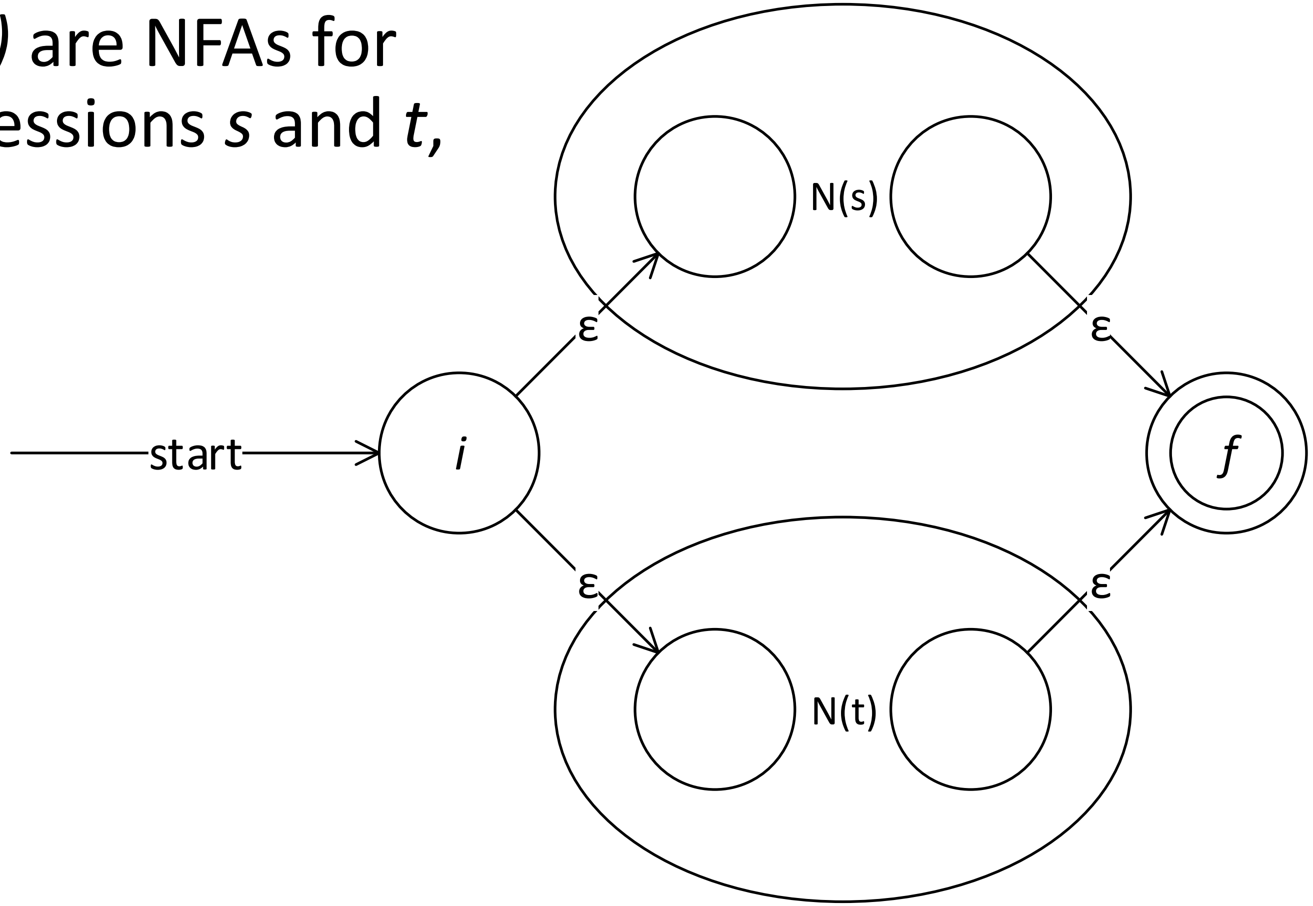
Construction of an NFA: expression a in Σ

- For regular expression $r = a$ (in Σ)



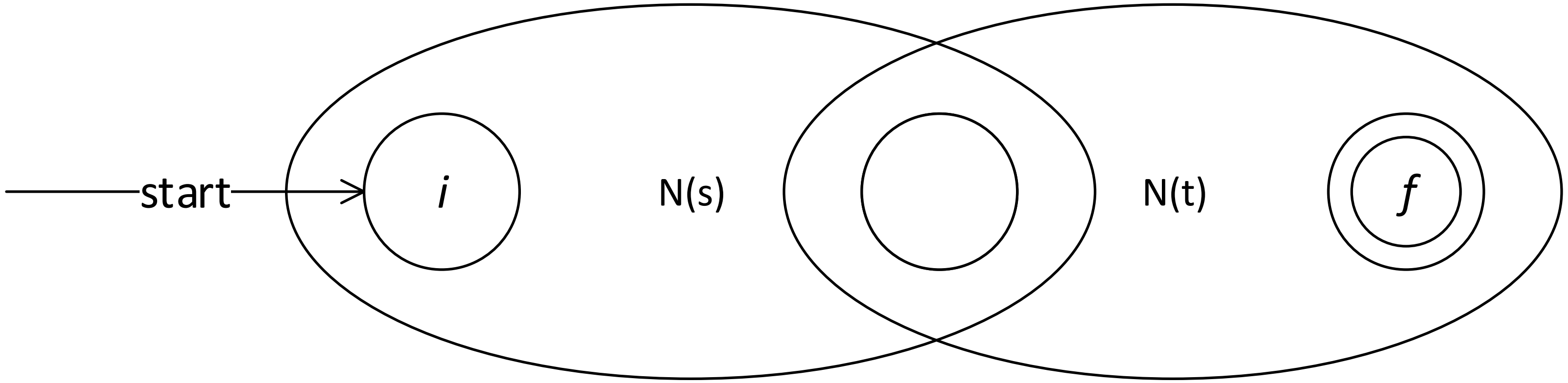
Construction of an NFA: expression with union (that is, |)

- For regular expression *r* = *s* | *t*
- *N(s)* and *N(t)* are NFAs for regular expressions *s* and *t*, respectively



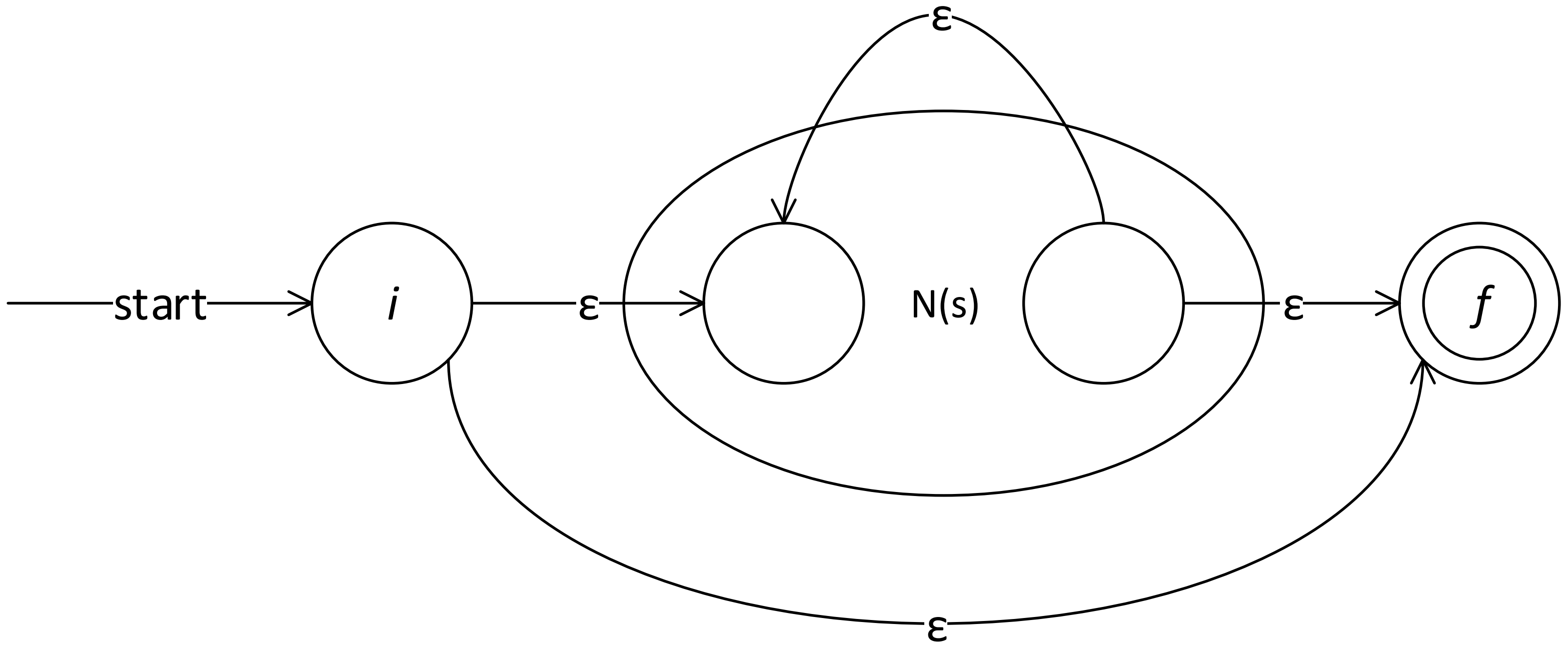
Construction of an NFA: expression with concatenation

- For regular expression *r* = *st*
- *N(s)* and *N(t)* are NFAs for regular expressions *s* and *t*, respectively



Construction of an NFA: expression with closure (that is, *)

- For regular expression *r* = *s**
- *N(s)* is an NFA for regular expression *s*



Subset Construction of a DFA from an NFA

- We refer to the NFA as N and to the DFA as D
- D’s states will be called *Dstates*
- D’s transitions will be encoded in a transition table *Dtran*
- Each state of D is a *set* of NFA states

Subset Construction of a DFA from an NFA

- s refers to a single state of N
- T refers to a set of states of N

- ε-closure(s) is the set of NFA states reachable from NFA state s on ε-transitions alone
- ε-closure(T) is the set of NFA states reachable from some NFA state s in set T on ε-transitions alone
- move(T, a) is the set of NFA states to which there is a transition on input symbol a from some state in T

Computing **ε**-closure(T)

```
push all states of T onto stack;
initialize ε-closure(T) to T;
while ( stack is not empty ) {
  pop t, the top element, off stack;
  for ( each state u with an edge from t to u labeled ε )
    if ( u is not in ε-closure(T) ) {
      add u to ε-closure(T);
      push u onto stack;
    }
}
```

Subset construction

```
initially, ε-closure(s0) is the only state in Dstates, and it is unmarked;
while ( there is an unmarked state T in Dstates ) {
  mark T;
  for ( each input symbol a ) {
    U = ε-closure(move(T, a));
    if ( U is not in Dstates)
      add U as an unmarked state in Dstates;
    Dtran[T, a] = U;
  }
}
```

Example

- Construct an NFA from a regular expression

- Construct a DFA from an NFA